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Solution Verification of Anomalous Waves in Nonideal Gases

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Solution verification methods for anomalous waves in inviscid and viscous van der Waals gases are presented. Anomalous waves are admissible in a single gas phase material when isentropes are concave, rendering the sound speed to have the unusual feature of decreasing with increasing density. The anomalous waves considered include rarefaction shocks and continuous compression fans. A previously known exact solution of inviscid continuous fans with a van der Waals equation of state is applied to anomalous waves. An exact solution for viscous shocks in an ideal gas is described and utilized for verification of the viscous numerical solutions. Solutions and simulations of viscous and inviscid van der Waals gases in shock tubes are presented with both conventional and anomalous waves. Shock tube solutions are used for verification of numerical simulations. Highly resolved viscous solutions are obtained with a simple explicit Euler time advancement scheme coupled with a second-order central spatial discretization. Inviscid simulations are performed with a third-order Runge–Kutta method in time and a fifth-order mapped weighted essentially nonoscillatory (WENO5M) discretization. The WENO5M method is novelly supplemented with a global Lax–Friedrichs flux-splitting in space, as local flux-splitting methods fail when changes in the sound speed are nonmonotonic. [DOI: 10.1115/1.4065834]

1 Introduction

Verification methods for numerical simulations of compressible flows are often restricted to systems modeled with the ideal gas equations; these yield classical compression shocks and rarefaction fans. Our focus will be on the verification of numerical methods when anomalous waves are present in viscous and inviscid nonideal gases. The anomalous waves in question refer to discontinuous rarefactions, continuous compressions, and composite waves. Although anomalous behavior may occur under a variety of conditions, we are interested in anomalous waves that form in a single gas phase material when isentropes in the pressure-specific volume plane are concave. When this occurs, the sound speed decreases as density increases, allowing for the formation of anomalous waves. When working with materials that meet the conditions for anomalous wave formation, it is important to ensure that numerical methods correctly predict the anomalous behavior. In the case of anomalous shock waves, the second law of thermodynamics may not be sufficient to distinguish correct inviscid shock solutions, and viscous shock solutions are required. We will utilize existing exact solutions for waves in inviscid van der Waals gases to construct analytical solutions for shock tubes. These inviscid solutions are also compared to numerical simulations of shock tubes containing a viscous van der Waals gas. These solutions will be used to demonstrate solution verification with nonideal gases and anomalous waves. The results presented here are an extension of the inviscid study of Pielemeier and Powers [1]. This work includes more detailed verification and an extension to viscous solutions.

We present a short review of convexity, as it is the cornerstone for the admission of anomalous waves. Defined on an interval $[v_1, v_2]$, a real valued function is convex if the line segment connecting any two points on the curve lies entirely above the curve between the two points. This is equivalent to the epigraph of the function being a convex set, with the epigraph defined as the set of points on or above the graph of a function. If this condition is not met, the epigraph is not a convex set, and the function is nonconvex. Example isentropes are shown in Fig. 1. The shaded areas represent the epigraphs; the epigraph in Fig. 1(*a*) forms a convex set, and in Fig. 1(*b*) a nonconvex set. The convexity of a function can also be determined by the second derivative of the function, for which positive and negative values correspond to convex and concave functions, respectively; this will be our primary method of describing convexity. If the second derivative of the function everywhere on the domain is positive semidefinite, then the function is convex. If the second derivative is everywhere negative on the domain, the function is concave. Following the terminology of Wendroff [2,3] and Heuzé et al. [4], if the second derivative is both positive and negative within the domain, we will refer to the function as nonconvex on the domain.

The first studies on the existence and theory of anomalous waves were done by Bethe [5], Zel'dovich [6], and later Thompson [7]. Bethe and Zel'dovich established that given a particular set of conditions, anomalous waves may occur in gases near the critical point of the thermodynamic vapor dome. In this anomalous region, sometimes called the Bethe–Zel'dovich–Thompson (BZT) region, the isentropes in the pressure-specific volume plane are concave, and discontinuous rarefaction shocks and continuous compression fans are admissible. Thompson [7] gave the dimensionless fundamental derivative as an indicator of the convexity of isentropes, the sign of which is determined by a second derivative of pressure with respect to specific volume. Positive values of this fundamental derivative indicate convex isentropes, and negative values indicate concave isentropes. Lambrakis and Thompson [8]

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